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ABSTRACT

The processes and strategies used by Finnish second graders in solving verbal multiplication and division tasks were investigated, and the relationship of these processes and strategies to Piagetian abilities, memory capacity skills, and rational number concepts was charted. Three categories of strategies were classified: operations based on external aids, short mental operations based on number listing in certain intervals, and operations based on knowledge or derived knowledge. The division strategies relying on the use of external aids were further classified in terms of measurement or partition situations. Tests designed by Piaget to measure the developmental levels of multiplication and division operations were found to be rather remote from the processes and strategies used on multiplication and division tasks at school. This finding, plus the results on memory capacity and understanding of rational number, are discussed in detail in this English summary, which includes a list of references. (MNS)

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RATKAISUPROSESSIT JA STRATEGIAT PERUSTAVISSA SANALLISISSA KERTO- JA JAKOLASKUTEHTÄVISSÄ

Tutkimus kerto- ja jakolaskutaitojen yhteyksistä Piaget-kykyihin, muistikapasiteettiin, lukujono-taitoihin sekä rationaalilukukäsitteeseen

ENGLISH SUMMARY

Processes and Strategies in Solving Elementary Verbal Multiplication and Division Tasks: Their Relationship with Piaget Abilities, Memory Capacity Skills and Rational Number

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ENGLISH SUMMARY

Processes and Strategies in Solving Elementary Verbal Multiplication and Division Tasks: Their Relationship with Piaget Abilities, Memory Capacity Skills, and Rational Number.

This study, the third stage of a longitudinal study, investigated the processes and strategies used by second-graders in solving elementary verbal multiplication and division tasks, and sought further to chart the relationship of these processes and strategies with Piaget abilities, memory capacity skills, and rational number. It was possible to classify the strategies employed into three categories on the basis of the degree of internalisation and abbreviation as follows: a) operations based on external aids, b) short mental operations based on number-listing in certain intervals, and c) operations based on knowledge or derived knowledge ($t < 2s.$). Elementary division strategies relying on the use of external aids were further classifiable according to whether the task involved measurement or partition. With measurement tasks the pupils immediately proceeded to arrange the given set of blocks into groups on the size indicated by the divisor then counted or recognised directly the number of groups thus formed. In the case of partition division tasks, on the other hand, the most common strategy was "try and see". In the other words the pupil tried with a certain size of group. If the number of groups turned out to be the same as the divisor, the answer could be given. If not, the trial and error procedure continued. It is to be noted that this strategy demands that the divisor be kept in mind throughout the processing. From this point of view partition division tasks are more demanding than measurement division tasks, where the size of group to be formed, or, in the case of a mental arithmetic strategy, the number from which to start counting, and by how much, is known from the start. The other strategy used on partition tasks was "give out one at a time" or a one-one strategy. This strategy, prominent in Piaget's experimental designs, was employed by only three to five pupils depending on the partition task. This finding is consistent with earlier studies by the author

concerning the formation of equivalence (Keranto 1981, 1983a). It is not surprising, therefore, that the present study yielded the following results. Both the theoretical and the experimental investigations established a close connection between the strategies used on multiplication and division tasks and the more demanding kind of sequence skills. Indeed it was found in regression analyses in the experimental section that number listing up or down from a given number by a certain number (of number words) together with number-listing in certain intervals accounted for 72% of the variance on multiplication strategies. The corresponding figures with relation to measurement and partition division were 62% and 40% respectively. The results are consistent with those obtained in the two earlier stages of the longitudunal study indicating close relationships between addition and subtraction strategies and sequence skills (Keranto 1983 a,b). The tests designed by Piaget to measure the developmental level of multiplication and division operations emphasizing prenumerical operations and partition division were found in this study to be rather remote from the processes and strategies used on elementary multiplication and division tasks at school. This conclusion was reached on both theoretical and experimental grounds. Indeed strategies bound to Piagetian multiple correspondence and the one-one strategy, together with transitive judgement and conservation, accounted for only 12-14% of strategies used on ordinary multiplication and division tasks. This result is again in keeping with earlier studies by the author charting the relationships between Piaget abilities and processes and strategies used in solving elementary arithmetic tasks (Keranto 1981, 1983 a).

This situation is also interesting from an epistemological point of view. The tests designed by Piaget to measure multiplication and division operations show that the purpose was also to seek to explain the development of the concept of number from a markedly cardinal, prenumerical operations point of view. If one-one correspondence could be seen to be the basis of the concept of number in mathematics, could it not also be the basis of the mental development of this concept?

But as was demonstrated in the theoretical section, this approach is not the only possible one. In mathematics an equally secure point of departure is Peano's axiomatic system which highlights the close connection between addition and multiplication operations. In fact, multiplication in this system is based on addition and thereby on the immediate-successor relationship of numbers. From the cardinal viewpoint this connection between addition and multiplication is not explicitly apparent. The findings of the present study indicate that the development of the concept of number, in respect of multiplication and division operations, too, can better be sought in number-listing and counting than in pre-numerical operations. It would appear that the individual's mental development of the concept of number is in line with a theoretical mathematical viewpoint, as long as this is not limited in too one-sided a manner to one theory of the foundation of number as Piaget seems to have done (cf. Beth & Piaget 1966, Brainerd 1978,1979, Keranto 1981,1983 a).

What, then, is the relationship of memory capacity to performance on multiplication and division tasks? Earlier studies indicated that memory capacity depended on the content under observation (cf. Keranto 1983 a,b). This appeared to be the case here, too. From earlier IPC test batteries an IPC measure was developed which closely involved measurement division. The child had to arrange a random display of fifteen blocks into groups of three while at the same time trying to hold given numbers in memory. It was found that IPC capacity thus measured correlated to some degree with the following variables: measurement division (0.43**), number listing in certain intervals (0.51**), multiplication (0.38*) and Piaget's multiple correspondence (0.48**). The correlations were of the same order as those obtained in earlier measurements between IPC and arithmetical operations. The reason for this in the present study is probably that all the second-graders measured were at the two-bit storage level and most at the three-bit level. In other words most of the pupils were able to hold in store (for example by mentally repeating) three number words while grouping blocks into threes. Logically, this size of

memory capacity ought to be sufficient to allow the information required in multiplication and division tasks to be kept in mind. This was reflected in the rather low correlations obtained.

The understanding of rational number was investigated in this study in contents involving ratios, fractions and partition division. The ratio test used was one developed by Noelting to measure proportion judgements in connection with a juice and water mixture (Noelting 1980 a,b). The result found was literally a complete hierarchy which corresponded to the levels of development put forward by Noelting. The average ages at which each level is achieved as presented by Noelting were also in line with the results. Three pupils had achieved the level of "higher concrete operations", i.e. were competent on the tasks 1:2 vs. 2:2 and 4:2 vs. 2:1. The majority (20/36) were on the level of "lower concrete operations", i.e. were competent on the tasks 1:1 vs. 2:2 and 2:2 vs. 3:3. The rest were on the level of "high intuition",i.e. were competent on the tasks 2:1 vs. 3:3 and 2:2 vs. 3:4 and tasks prior to these. The logical relationships between this ratio test and multiplication and division operations were found to be few if the pupils were no further advanced than the level of concrete operations. Only at Noelting's formal operations level do multiplication and division skills play a significant role in performance on such tasks. This was evident in the low figures for explanatory power obtained in the experimental results (22%). A similar situation obtained in relation to sequence skills, Piaget abilities and IPC abilities (0-12%). In other words operations involved in the ratio test were quite "separate" from both multiplication and division skills and Piaget abilities, IPC abilities and sequence skills.

The understanding of rational number in contents involving fractions was investigated be means of a development of Noelting's test using a surface model. The pupil was required to compare certain segments of equal-sized "cakes" which had been cut into ceratin equal-sized pieces. The segments for comparison were chosen so as to correspond to the number ratios in

Noelting's test such that if a:b vs. c:d, then $\frac{a}{a+b}$ vs. $\frac{b}{c+d}$. This test did not reveal such a clear hierarchy as the Noelting test. One problem lay in the presentation of the pictures (two "cakes"). The segments relating to the numbers were coloured in the pictures, so that in many cases pupils made visual comparisons. Scoring thus had to be based on their original mental answers (the pictures were shown only after a mental attempt). Hierarchy values were: CR=0.96, MMR=0.88 and PPR=0.67, with only the following tests being used: 4/5 vs. 1/5, 3/4 vs. 2/4, 1/2 vs. 2/4, 2/4 vs. 3/6, 1/3 vs. 2/6 and 4/6 vs. 2/3. The "higher concrete operations" level or stage was achieved by 5 pupils. Otherwise the frequency of the concrete operations performance level was very low (8 pupils) compared with performance on the Noelting test. This is probably due to the criterion of mental performance. As with the Noelting test, none of the second-grade pupils had achieved the "formal operations" level. Thus both from the theoretical and the experimental point of view the degree of connection between the fraction number test and multiplication and division skills, Piaget abilities and sequence skills is not very high. In fact the levels of prediction were very low at 14-26%, where multiplication skills were the highest and Piaget abilities the lowest predictors. The correlation between the ratio number and fraction number tests was 0.45. This suggests that at the second-graders' level the operations in question are still somewhat "separate" from eachother. Presumably the connection between them becomes stronger at higher school levels. This remains to be seen in the future stages of the longitudinal study.

At some stage the pupil has to come understand the fraction a/b as a ratio of two numbers a:b. Touching on this the present study investigated judgements of proportion in partition division situations. This was done by asking pupils to decide if the girls or the boys got more, or the same amount, of chocolate shared out. The "chocolate bars" were ready divided into certain numbers of pieces. The pieces were of the same size but their number and the number of divisors (recipients) varied corresponding in part with the stages and levels in

Noelting's test. Tasks on the "intuitive" level did not necessarily require a process of partition as judgement could be made directly by comparing the number of pieces or divisors (recipients). As expected, this test was more closely connected than the ratio and fraction tests both with multiplication and division skills and with Piaget abilities ans sequence skills. The best predictor here proved to be Piaget abilities, which relate logically precisely to the kind of task involving comparison of numbers. The predictive power of Piaget abilities was 53%, while the corresponding figures for sequence skills and multiplication and division skills were 26% and 36% respectively. Taken together, the three variables predicted 64% of proportion judgements involving partition division. The correlations between this "pieces of chocolate" test and the "juice" and "cake" tests were, as expected, rather low ar 0.37 and 0.40 respectively. This, too, points to the "separateness" in the child's mind of operations and judgements involving rational number. In other words, such judgements are dependent on contents. It is for a further stage of research to find out how these operations are linked to the ability to find a fractional post of certain number or quantity. It is reasonable to assume this will provide an answer to the question of the stage at which the pupil finds the connection between the fraction a/b and the ratio a:b. This would be an important thing to know from the teaching point of view, too, as such knowledge might serve to make easier many of the formal operations involving the concept of fraction and later to expand rational number to real number via decimals. It would also be important to investigate the relationship between the areas examined in this study and the understanding of rational number as decimal number. Interest would then be focussed at the same time on how the child in practice learns the ideas of the decimal system we use and what conscious connection there is between this and the acquisition of decimal number. Logically a central place in the learning of decimal number is occupied by the transfer and generalisation of algorithms of our numbering system to situations involving fraction numbers. How this takes place in the mind of the child would be an interesting and important object of research which it is not

possible to go into further in the present study. It is hoped, however, that this condensed report will offer ideas and points of departure for research into the development of basic mathematical concepts and their interconnections, and for the application of results to practice in the school.

REFERENCES

Ausubel, D.P. 1963. The Psychology of Meaningful Verbal Learning. N.Y.: Grune and Stratton

Beth, E.W. & Piaget, J. 1966. Mathematical Epistemology and Psychology. Dordrecht, Holland: Reidel

Brainerd, C.J. 1978. Piaget's Theory of Intelligence. New Jersey: Prentice-Hall

Brainerd, C.J. 1979. The Origins of the Number Concept. N.Y.: Praeger

Cantor, G. 1962. Gesammelte Abhandlungen mathematischen und philosophischen Inhalts. Herausgegeben von Ernst Zermelo, Nebst einem Lebenlauf Cantors von Adolf Fraenkel. Hildesheim: Georg Olms

Case, R. 1972. Validation of a neo-Piagetian mental capacity construct. Journal of Experimental Child Psychology 14: 287-302

Case, R. 1980. The underlying mechanism of intellectual development. In J.R. Kirby & Biggs, J.B. (Eds.), Cognition, Development and Instruction. N.Y.: Academic Press

Chapman, R.H. 1975. The development of Children's understanding of proportions. Child Development 46: 141-148

Copeland, R.W. 1979. How children learn mathematics. N.Y.: Macmillan

Falk, R.R. & Levin, I. 1980. A potential for learning probability in young children. Educational Studies in Mathematics 11: 181-204

Flavell, J.H. 1963. The development psychology of Jean Piaget. Toronto, London, Melbourne: Van Nostrand

Freudenthal, H. 1973. Mathematics as an Educational Task. Dordrecht, Holland: Reidel

Fuson, K. & Hall, J.W. 1982. The Acquisition of Early Number Word Meanings: A Conceptual Analysis an Review. In H.P. Ginsburg, (ed.), Children's Mathematical Thinking. N.Y.: Academic Press

Fuson, K. & Richards, J. & Briars, D.J. 1982. The acquisition and elaboration of number word sequence. In C. Brainer (ed.), Progress in Cognitive Development. Children's Logical and Mathematical Cognition (Vol 1), N.Y.: Springer-Verlag

Greeno, J.G. 1976. Cognitive Objectives of Instruction: Theory of Knowledge for Solving Problems and Answering Questions. In D. Klahr, (ed.), Cognition and Instruction. Hillsdale, New Jersey: Lawrence Erlbaum

Guttman, L. 1944. Basis for scaling qualitative data. Amer. Sociol. Rev. 77: 139-150

Hiebert, J. & Carpenter, T.P. 1982. Piagetian tasks as readiness measures in mathematics instruction: a critical review. Educational Studies in Mathematics 13: 329-345

Hiebert, J. & Carpenter, T. & Moser, J.M. 1982. Cognitive Development and Children's Solutions to Verbal Arithmetic Problems. Journal for Research in Mathematics Education 13: 83-98

Inhelder, B. & Piaget, J. 1958. La genése des structures logiques e lémentaires. Delachaux et Niestlé, Neuchatel. Translated as The Early Growth of Logic in the Child: Classification and Seriation. London: Routledge and Kegan Paul

Karplus, R. & Peterson, F.W. 1970. Intellectual development beyound elementary school II; Ratio, a survey. School-Science and Mathematics 70: 813-820

Keranto, T. 1979. Lapsen lukukäsitteen kehittymisen ja kehittämisen matemaattisdidaktinen näkökulma. Hämeenlinna. Tampereen yliopiston Hämeenlinnan OKL:n julkaisuja, No 2

Keranto, T. 1981. Lukukäsitteen kehittyminen ja kehittäminen: matemaattis-loogiset perusteet ja luvun kognitiivinen rakentuminen. Acta Universitatis Tamperensis. ser.A vol. 125. University of Tampere.

Keranto, T. 1983 a. Matemaattiset ajatteluprosessit ja strategiat: yhteydet yksilön tiedolliseen rakentumiseen ja informaation käsittelykapasiteettiin. Hämeenlinna. Tampereen yliopiston Hämeenlinnan OKL:n julkaisuja, No 8

Keranto, T. 1983 b. Aritmeettiset prosessit ja strategiat: yhteydet matematiikan opintomenestykseen, opiskelumotivaatioon ja asenteisiin. Hämeenlinna. Tampereen yliopiston Hämeenlinnan OKL:n julkaisuja, No 10

Kieren, T. 1976. On the Mathematical, Cognitive and Instructional Foundations of Rational Numbers. In R. Lesh (ed.), Number and Measurement: Papers from Research Workshop, ERIC/SMEAC, Ohio State University, Columbus

Landau, E. 1960. Grundlagen der Analysis. N.Y.: Chelsea Publishing Combany

Leino, J. 1977. Matematiikan didaktiikka 1. Helsinki: Kirjayhtymä

Leino, J. 1981. Psychometric Test Theory and Cognitive Processes: A Theoretical Scrutiny and Empirical Research. Research Bulletin No 57. Institute of Education, University of Helsinki

Leino, J. 1982. Kognitiivisten prosessien tutkimus matematiikan opetuksessa ulkomailla. Julkaisussa Kognitiiviset prosessit ja matematiikan opetus. Kasvatustieteiden tutkimuslaitoksessa 8.-9.2.1982 järjestetyn matematiikan opetuksen tutkijaseminaarin raportti. Toimittaja P.Kupari. KTL:n Selosteita ja Tiedotteita 199

Noelting, G. 1980 a. The development of proportional reasoning and the ratio concept. Part 1 - Differentiation of stages. Educational Studies in Mathematics 11: 217-253

Noelting, G. 1980 b. The development of proportional reasoning and the ratio concept. Part 2 - Problem-structure at successive stages: problem-solving strategies and mechanism of adaptive restructuring

Payne, J.N. 1976. Review of Research on Fractions. In R.Lesh (ed.), Number and Measurement: Papers from Research Workshop, ERIC/SMEAC, Ohio State University, Columbus

Pehkonen, E. 1978. Matematiikan peruskurssi 1 korkeakouluja varten. Helsinki: Kirjayhtymä

Piaget, J. 1970. Genetic Epistemology. N.Y.: Columbia University Press

Piaget, J. 1971. The Child's Conception of Movement and Speed. N.Y.: Ballantine Books

Piaget, J. 1952. The Child's Conseption of Number. N.Y.: Humanities

Piaget, J. & Inhelder, B. 1951. La genese de l´idee de hasard chez l´enfant, Press Universitaires de France, Paris. Translated as: The Origin of the Idea of Chance in Children. N.Y.: Norton, 1975

Piaget, J., Inhelder, B. & Szeminska, A. 1960. The child's conception of geometry. N.Y.: Basic Books

Resnick, L.B. 1976. Task Analysis in Instructional Design: Some Cases from Mathematics. In D.Klahr (ed.), Cognition and Instruction. Hillsdale, New Jersey: Lawrence Erlbaum

Siegel, S. 1958. Nonparametric statistics for behavioral sciences. N.Y.: Mc Graw

White, W. & Saltz, E. 1957. Measurement of reproducibility. Psychological Bulletin 54,2: 81-99